Reflection Coefficient of the Entire System (Eq 2)

$$\tilde{r} = \frac{[\tilde{r}_p + \tilde{r}_s e^{-i\Delta}]}{2}$$

Eq (1) in the paper. The tilde-hat indicates complex numbers (i.e. the amplitude reflection coefficients are complex quantities, since the metal's refractive index is a complex value).

$$|r|^2 = \tilde{r}\tilde{r}^* = \frac{[\tilde{r}_p + \tilde{r}_s e^{-i\Delta}]}{2} \frac{[\tilde{r}_p^* + \tilde{r}_s^* e^{+i\Delta}]}{2}$$

The superscript asterisk indicates a complex conjugate (i.e. multiplying all imaginary quantities by -1).

A complex number multiplied by its complex conjugate is the magnitude squared.

Multiply out the numerator.

$$= \frac{\tilde{r}_p\tilde{r}_p^* + \tilde{r}_p\tilde{r}_s^* e^{+i\Delta} + \tilde{r}_p^*\tilde{r}_s e^{-i\Delta} + \tilde{r}_s\tilde{r}_s^* e^{i(\Delta-\Delta)}}{4}$$

$$= \frac{\tilde{r}_p\tilde{r}_p^* + \tilde{r}_p\tilde{r}_s^* e^{+i\Delta} + \tilde{r}_p^*\tilde{r}_s e^{-i\Delta} + \tilde{r}_s\tilde{r}_s^*}{4}$$

$$= \frac{\tilde{r}_p\tilde{r}_p^* + \tilde{r}_s\tilde{r}_s^* + \tilde{r}_p\tilde{r}_s^* e^{+i\Delta} + \tilde{r}_p^*\tilde{r}_s e^{-i\Delta}}{4}$$

$$= \frac{|r_p|^2 + |r_s|^2 + \tilde{r}_p\tilde{r}_s^* e^{+i\Delta} + \tilde{r}_p^*\tilde{r}_s e^{-i\Delta}}{4}$$

Reflection Coefficient of the Entire System (Eq 2)

$$|r|^2 = \tilde{r}\tilde{r}^* = \frac{|r_p|^2 + |r_s|^2 + \tilde{r}_p\tilde{r}_s^* e^{+i\Delta} + \tilde{r}_p^*\tilde{r}_s e^{-i\Delta}}{4}$$

$$\tilde{r}_p = r_{p\_real} + i r_{p\_imag} = r_p e^{+i\phi_p}$$

$$\tilde{r}_p^* = r_{p\_real} - i r_{p\_imag} = r_p e^{-i\phi_p}$$

$$\tilde{r}_s = r_{s\_real} + i r_{s\_imag} = r_s e^{+i\phi_s}$$

$$\tilde{r}_s^* = r_{s\_real} - i r_{s\_imag} = r_s e^{-i\phi_s}$$

Magnitude

Phase multiplied by *i*

Represent the complex reflection coefficients ($\tilde{r}$) explicitly in terms of their magnitude (no tilde):

$$r = \sqrt{r_{real}^2 + r_{imag}^2}$$

and phase:

$$\phi = \tan^{-1}\left(\frac{r_{imag}}{r_{real}}\right)$$

Substitute the magnitude-phase representations of the complex amplitude reflection coefficients into the equation at the top of this slide.

$$|r|^2 = \tilde{r}\tilde{r}^* = \frac{|r_p|^2 + |r_s|^2 + r_p e^{+i\phi_p} r_s e^{-i\phi_s} e^{+i\Delta} + r_p e^{-i\phi_p} r_s e^{+i\phi_s}\, e^{-i\Delta}}{4}$$

$$|r|^2 = \tilde{r}\tilde{r}^* = \frac{|r_p|^2 + |r_s|^2 + r_p r_s e^{+i(\phi_p - \phi_s + \Delta)} + r_p r_s e^{-i(\phi_p - \phi_s + \Delta)}}{4}$$

Reflection Coefficient of the Entire System (Eq 2)

$$|r|^2 = \tilde{r}\tilde{r}^* = \frac{|r_p|^2 + |r_s|^2 + r_p r_s e^{+i(\phi_p - \phi_s + \Delta)} + r_p r_s e^{+i(\phi_p - \phi_s + \Delta)}}{4}$$

The last two terms in the numerator are the same, except for the sign on the exponential.

Use the Euler equation (trig identity):

$\cos\theta = \frac{e^{ix} + e^{-ix}}{2}$ where $x = \phi_p - \phi_s + \Delta$

to obtain:

$$|r|^2 = \tilde{r}\tilde{r}^* = \frac{|r_p|^2 + |r_s|^2 + 2r_p r_s \cos(\phi_p - \phi_s + \Delta)}{4}$$

Define $\phi = \phi_p - \phi_s$

$$|r|^2 = \tilde{r}\tilde{r}^* = \frac{|r_p|^2 + |r_s|^2 + 2r_p r_s \cos(\phi + \Delta)}{4}$$

Reflection Coefficient of the Entire System (Eq 2)

$$|r|^2 = \tilde{r}\tilde{r}^* = \frac{\left|r_p\right|^2 + |r_s|^2 + 2r_p r_s \cos(\phi + \Delta)}{4}$$

The last two terms in the numerator are the same, except for the sign on the exponential.

The lowercase 'r' refers to the optical amplitude reflection coefficients.
Uppercase 'R' are used to refer to optical power reflection coefficients.

$$\mathrm{R} = |r|^2$$
$$R_p = \left|r_p\right|^2$$
$$R_s = |r_s|^2$$

When these representations are substituted into the equation at the top of this slide,

$$R = |r|^2 = \frac{R_p + R_s + 2\sqrt{R_p R_s}\cos(\phi + \Delta)}{4}$$

It is the square root that is missing from Eq. 2 in the paper.