**Pancharatnam's phase : An example of geometric phase of light**

The vector nature of electromagnetic waves is reflected in their polarization properties. Polarization, like the dynamical phase, represents a physical degree of freedom of an electromagnetic wave. This means that during the propagation of an electromagnetic wave through material media, the evolution of both the dynamical phase and polarization must be considered. An electromagnetic plane wave of angular frequency $\omega$ propagating in the $z$-direction can be represented by its electric field

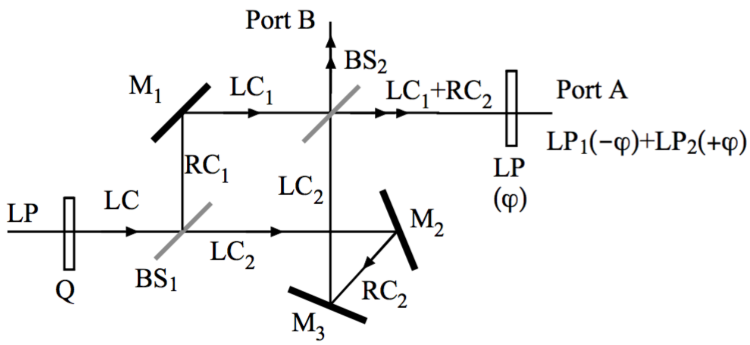
$$\boldsymbol{E}(\boldsymbol{r}, t) = \left(\boldsymbol{e}_x \, E_x + \boldsymbol{e}_y \, E_y e^{i\delta}\right) e^{i(\phi(z,t)+\phi_o)} \, , \tag{1}$$

where the dynamical phase

$$\phi(z, t) = (n\omega \, z/c) - \omega t \, , \tag{2}$$

and $\phi_o$ is a constant angle representing the initial or reference phase, $n$ is the refractive index of the medium, and $c$ is the speed of light. The polarization state (orientation of the electric vector) of the wave is determined by the relative magnitudes of the $x$ and $y$ components of the electric vector, $E_x$ and $E_y$, respectively, and their relative phase difference $\delta$. The polarization state of the wave and its evolution can be described as a point and its trajectory on the surface of Poincare sphere or as a Jones vector and its transformation.

The evolution of the dynamical phase during propagation is governed by the wave equation and is well known [See Eq. (2) above]. If the polarization state of the wave changes during propagation, the wave may acquire an extra contribution to its phase, which is in addition to the change in its dynamical phase. This phase contribution was first discussed by Pancharatnam [1] and is referred to as Pancharatnam's phase of light. It is an example of geometric or topological phase of light. Under certain conditions, Panchratnam's phase may be related to Berry phase [2] although the former predates the latter. During the propagation, wave polarization evolves and, in general, the initial and final states of wave polarization may be different. Pancharatnam devised a way of comparing the phases of two coherent light waves in different states of polarization. He showed that Pancharatnam's phase of light depends on the trajectory of wave polarization on Poincare sphere. For a cyclic evolution of polarization, the trajectory of polarization on Poincare sphere is a closed curve. In this case, Pancharatnam's phase equals half the solid angle subtended at the origin by the area enclosed by the closed curve on Poincare sphere. Pancharatnam's phase can also be calculated by tracing the evolution of wave polarization using Jones matrix description of wave polarization.




There are several ways of observing Pancharatnam's phase of light. They all involve an interference experiment. This is hardly surprising, since we are concerned with an observation of wave phase change, which can be measured by comparing the phase of the initial and final waves in an interference experiment. Figure here shows a modified Mach-Zehnder interferometer into which, say, a left circularly polarized light wave is injected. The polarization in the two arms of the interferometer evolves differently as the waves travel to meet at the second beam splitter $BS_2$. To see this we recall that upon reflection, a left circularly polarized (LC) light wave is turned into a right circularly polarized (RC) light wave. Then the polarization along the upper path (subscript 1) evolves according to $LC \rightarrow RC_1 \rightarrow LC_1 \rightarrow LC_1$ as it emerges from beam splitter $BS_2$. Along the lower path (subscript 2) it evolves according to $LC \rightarrow LC_2 \rightarrow RC_2 \rightarrow LC_2 \rightarrow RC_2$ as it emerges from $BS_2$. If a screen is now placed before the linear polarizer LP at port A, no interference fringes will be observed since the two interfering beams have orthogonal polarizations – $LC_1$ is left circularly polarized and $RC_2$ is right circularly polarized. On the other hand, if a screen is placed after the linear polarizer LP, its transmission axis at an angle $\varphi$ to the $x$-axis, interference fringes are observed. Moreover, as the linear polarizer LP is rotated, fringes shift and as many fringes can be shifted as one likes by continuing to rotate LP in the same sense. The direction of fringe shift is reversed if the sense of rotation of LP is reversed. All through this, as LP is rotated, the path difference, if any, between the two arms of the interferometer does not change. Thus the contribution of dynamical phase to fringe shift is zero and the only contribution to phase change and, therefore, fringe shift comes from

Pancharatnam's phase.

For the case shown here, since only Pancharatnam's phase (due to the evolution of polarization) contributes to change in phase difference, we can calculate it by using Jones vector formalism. Jones vectors of $LC_1$ and $RC_2$ waves are

$$\langle \mathrm{LC}_1| = \begin{pmatrix} 1 \\ -i \end{pmatrix} , \qquad \langle \mathrm{RC}_2| = \begin{pmatrix} 1 \\ i \end{pmatrix} . \tag{3}$$

After passing through the linear polarizer LP at an angle $\varphi$ to the $x$-axis, polarization states of the transmitted waves are

$$\langle \mathrm{LP}_1| = \begin{pmatrix} \cos^2\varphi & \sin\varphi\cos\varphi \\ \sin\varphi\cos\varphi & \sin^2\varphi \end{pmatrix} \begin{pmatrix} 1 \\ -i \end{pmatrix} = \begin{pmatrix} \cos\varphi \\ \sin\varphi \end{pmatrix} e^{-i\varphi} , \tag{4}$$

$$\langle \mathrm{LP}_2| = \begin{pmatrix} \cos^2\varphi & \sin\varphi\cos\varphi \\ \sin\varphi\cos\varphi & \sin^2\varphi \end{pmatrix} \begin{pmatrix} 1 \\ i \end{pmatrix} = \begin{pmatrix} \cos\varphi \\ \sin\varphi \end{pmatrix} e^{+i\varphi} . \tag{5}$$

Thus emerging from LP are two linearly polarized coherent beams (both polarized along the transmission axis of LP) but with a relative phase difference of $2\varphi$. The same result is obtained by tracing the polarization trajectories of the two waves on Poincare sphere. One finds that the magnitude of area enclosed[1] by the two polarization trajectories is $4\varphi$, half of which is Pancharatnam's phase $2\varphi$, in agreement with the result from Jones matrix formalism.

In this setup, Pancharatnam's phase depends on the angle $\varphi$ of orientation of LP and, therefore, changes as the linear polarizer is rotated. Moreover, the sign of Pancharatnam's phase depends on the sense in which linear polarizer is rotated. This property, responsible for the reversal of the direction of fringe shift with the reversal of the sense of rotation of LP, follows from the fact that in computing Pancharatnam's phase, the area on Poincare sphere must be treated as a vector quantity. So the sign of the solid angle subtended at the origin of Poincare sphere by the area enclosed by the closed polarization trajectory is determined by the sense (clockwise or counter-clockwise) in which the trajectory is traversed.

Polarization is a physical degree of freedom of an electromagnetic wave and like the dynamical phase of the wave, the evolution of wave polarization

[1]Both beams begin with the same initial polarization (LC) and end with the same final polarization LP (which is different from the initial polarization). Therefore, their polarization trajectories form a closed loop on Poincare sphere.

must be considered during propagation. Pancharatnam's phase is a fascinating manifestation of this additional degree of freedom associated with the evolution of wave polarization.


1. S. Pancharatnam, Proc. Ind. Acad. Science A44, 247 (1956) also in *Collected Works of S. Pancharatnam* (Oxford U. Press, Oxford 1975).

2. M. V. Berry, Proc. Roy. Soc. (London) 392, 45 (1984) and J. Mod. Optics **34**, 1401 (1987). Both papers are reprinted in *Geometric Phases in Physics*, Ed. A Shapere and F. Wilczek (World Scientific, Singapore, 1989). See also M. V. Berry in Physics Today **41**, 34 (1990).